HYPER VERSION

RECEIVED: *February 15, 2001*, ACCEPTED: *March 19, 2001*
REVISED: *March 13, 2001*

Mode interactions of the tachyon condensate in p-adic string theory

Joseph A. Minahan

Department of Theoretical Physics
Box 803, SE-751 08 Uppsala, Sweden
E-mail: joseph.minahan@teorfys.uu.se

ABSTRACT: We study the fluctuation modes for lump solutions of the tachyon effective potential in p-adic open string theory. We find a discrete spectrum with equally spaced mass squared levels. We also find that the interactions derived from this field theory are consistent with p-adic string amplitudes for excited string states.

KEYWORDS: Non-perturbative Effects, Bosonic Strings.

JHEP03(2001)028

Contents

1. Introduction

Sen's conjectures [1] concerning tachyon condensation and D-brane decay have been widely tested using cubic string field theory [2] or the superstring analog [3]. There have also been many promising attempts to find an exact verification using cubic string field theory [4, 5].

Field theory models of tachyon dynamics are also useful in understanding the qualitative picture of tachyon condensation. One such model was proposed in [6], which was constructed by looking for lump solutions with a discrete fluctuation spectrum. This model is in some sense a generalization of a purely cubic model [7] and was later shown to be the two derivative limit of boundary string field theory [8]–[11]. An analagous model was constructed for the superstring [12] and it too was later shown to be the two derivative limit of an action derived from boundary string field theory [13]

The authors of [9] pointed out that the model in [6] can be obtained as the $p \to 1$ limit of p-adic string theory. The p-adic string describes tachyon dynamics in a vastly simpler context than the ordinary bosonic string [14]. In particular, Ghoshal and Sen showed that the open string dynamics disappear at the stable vacuum. They also showed that the lump solutions of the effective field theory satisfy constant descent relations [14]. One can ask how other ideas of tachyon condensation can be tested using the p-adic string.

The p-adic string tree amplitudes are found by replacing the integrals over real numbers normally found in bosonic string theory, with integrals over p-adic numbers [15], [17]–[21]. The resulting amplitudes are much simpler than in bosonic string theory. For example, the N-point tachyon amplitude only has tachyon poles. Using these amplitudes, the authors of [20] were able to construct the exact tree-level action for the tachyon field. This action is non local, with a potential that is unbounded below, but with a local minimum. The equations of motion have non-trivial stationary classical solutions, where the tachyon field lies at the local minimum at spatial infinity. These solutions were later interpreted to be the analogs of D-branes [14].

Since the tachyon amplitude only has tachyon poles, one might think that a consistent p-adic string theory only needs the tachyon field. In fact, it is not clear that one can even put in many of the other fields familiar from bosonic string theory. For instance, there has been no successful attempt to put in gauge fields such that the resulting amplitudes are gauge invariant. Gauge invariance requires that the on-shell amplitudes be zero when a polarization vector ξ_μ is replaced by the momentum vector k_μ. In bosonic string theory, this works because the vertex operator for the gauge boson,

$$V = \zeta \cdot \partial_t X e^{ik\cdot X}\,, \tag{1.1}$$

where ∂_t refers to the tangential derivative along the boundary of the world-sheet, becomes a total derivative. However, for the p-adic numbers, there is no well-defined notion of a derivative operator [20], hence there is no guarantee that the amplitude is zero after p-adic integration.

For example, suppose one considers the correlation functions of the vertex operators in (1.1). In ordinary bosonic string theory, the correlator is not positive definite over the boundary of the string world-sheet; there are points where the integrand changes sign. One can define a sign operator for p-adic numbers, in fact there can be more than one such operator [20]. Hence, one could define the p-adic amplitudes by using the ordinary bosonic string correlators, replacing the real number coordinates on the boundary of the string world sheet with p-adic numbers and inserting one of the p-adic sign operators where appropriate. Unfortunately, this does not yield gauge invariant results, as one can easily check with the simplest non-trivial four-point amplitude which has two external tachyons and two external gauge fields.

Instead of showing this explicitly, let us give another argument why gauge fields are likely to be absent. In [20] it was shown how to insert Chan-Paton factors into the p-adic tachyon amplitudes by using a p-adic sign operator. In bosonic string theory, when Chan-Paton factors are inserted, non-abelian gauge poles appear in the tree amplitudes, since the tachyons have gauge indices. One can also show that these amplitudes are consistent with the tachyon-tachyon-gauge three point couplings. But in the p-adic amplitudes, even after inserting Chan-Paton factors, the only poles are tachyon poles. If a gauge vertex operator existed, then the three point coupling would be the same as for the bosonic string, since there is no integration over a number field. This would then lead to an inconsistency with the higher point amplitudes. Therefore, for the p-adic string, the Chan-Paton symmetries are global symmetries only.

Nevertheless, the existence of lump solutions must lead to other p-adic string states. If one examines the effective field theory around the lump solution, one finds fluctuation modes and the question is how do these modes fit into the p-adic string picture. The lowest such mode is a tachyon [14, 22], while the next mode is the scalar zero-mode [14].

In this paper we will show that the remaining modes lie in an infinite tower of states, with a discrete spectrum and evenly spaced mass squared levels, for any value of p. From the tachyon effective action, one can compute the effective action for the fluctuation modes. From this effective action one can directly compute tree-level scattering amplitudes.

In [23] it was conjectured that in bosonic string theory the fluctuations of the tachyon field about a codimension d lump solution correspond to the open string states

$$\prod_{i=1}^{d}\left(\alpha_{-1}^{i}\right)^{m_i}|0\rangle\,, \tag{1.2}$$

where the product is over the transverse directions to the brane. The m_i are non-negative integers and α_{-1}^{i} is a string oscillator transverse to the brane. One way to test the conjecture of [23] is to compute the amplitudes for the states in (1.2) using p-adic integration, and compare this to the amplitudes found from the p-adic effective action. We do this and find agreement between the two results.

In section 2 we study the fluctuation modes about the p-adic lump solutions and find their spectrum and compute their interactions. In section 3 we compute the p-adic string amplitudes for the vertex operators in (3.1). In doing this we keep track of combinatoric factors and simply borrow the results from [15], [17]–[21] for tachyonic amplitudes, without encountering any new types of p-adic integrals. In section 4 we give a brief discussion.

2. Mode interactions from the tachyon field theory

The starting point for this analysis is the tachyon effective action for the p-adic string [20]

$$\mathcal{S}=-\frac{1}{g^2}\frac{p^2}{p-1}\int d^Dx\left[\frac{1}{2}\phi p^{-\frac{1}{2}\Box}\phi-\frac{1}{p+1}\phi^{p+1}\right]. \tag{2.1}$$

While this action was derived for p a prime number, it appears that p can be continued to any positive real number. The equation of motion for the ϕ field is easily derived from (2.1) and is

$$p^{-\frac{1}{2}\Box}\phi=\phi^p\,. \tag{2.2}$$

The perturbative open string vacuum is at $\phi=1$, while the solution at the local minimum $\phi=0$ has no open string fluctuations, since all poles are absent. One advantage of studying the p-adic string is that the strong coupling problems that arise at the local minimum in boundary string field theory [24] are not a problem here[1] so long as p is an integer greater than 1 [5].

There are also non-trivial static solutions of (2.2). In particular, there are codimension d lump solutions [20, 14]

$$\phi(x_i) = \prod_{i=1}^{d} f(x_i)\,, \tag{2.3}$$

where $f(x)$ is the gaussian

$$f(x) \equiv p^{\frac{1}{2(p-1)}} \exp\left(-\frac{1}{2}\frac{p-1}{p\ln p}x^2\right). \tag{2.4}$$

This follows from the identity

$$\mathcal{A}\ f(x) = (f(x))^p\,, \qquad \mathcal{A} \equiv p^{-\frac{1}{2}\frac{\partial^2}{\partial x^2}}\,. \tag{2.5}$$

Plugging the solution in (2.3) back into (2.1) one can compute the tension of the lump solutions. In particular, one finds that the ratio of tensions satisfies the descent relation [14]

$$\frac{T_{D-d-1}}{T_{D-d-2}} = \left(\frac{2\pi p^{\frac{2p}{p-1}}\ln p}{p^2-1}\right)^{-1/2}\,. \tag{2.6}$$

Note that this ratio, as well as all tree amplitudes are invariant under $p \to 1/p$ [21]. One can show this by taking the equation of motion in (2.2) and substituting $\phi = \chi^{1/p}$. Then one finds the same equation for χ, but with p replaced by $1/p$.

Next consider the fluctuations about the lumps. For what follows, we will assume that $d = 1$, but the generalization for arbitrary d is straightforward. Hence, to quadratic order in the fluctuations, the action in (2.1) becomes

$$S_f = -\frac{1}{g^2}\frac{p^2}{p-1}\int d^{D-1}y\,dx\,\frac{1}{2}\widetilde{\phi}(y,x)\left[p^{-\frac{1}{2}\left(\Box_{\|}+\frac{\partial^2}{\partial x^2}\right)} - p(f(x))^{p-1}\right]\widetilde{\phi}(y,x)\,, \tag{2.7}$$

where y are the coordinates along the lump world-brane, $\widetilde{\phi}(y,x) = \phi(y,x) - f(x)$, and $\Box_{\|}$ is the d'Alembertian along the lump world-brane coordinates. To find the eigenmodes, we use the following identity

$$[\mathcal{A}, x] = -\ln p\frac{\partial}{\partial x}\ \mathcal{A}\,. \tag{2.8}$$

[1]Nevertheless, one could argue that the problem is alleviated by going to the weak string coupling limit. If the coupling is absorbed into the tachyon field ϕ, then the potential is $V(\phi) = -\frac{1}{4}\phi^2\ln(g^2\phi^2)$. Hence the mass of the field near the local minimum is roughly $-\frac{1}{2}\ln(g^2\phi^2)$ while the coupling terms have no g dependence. Hence, we can have a large mass and small couplings by keeping ϕ large, but $g\phi$ small.

Thus we find

$$\mathcal{A}\, x^n f(x) = p^n Q_n(x)(f(x))^p\,, \tag{2.9}$$

where $Q_n(x)$ is a polynomial of the form

$$Q_n(x) = x^n + C_{n-2}x^{n-2} + C_{n-4}x^{n-4} + \cdots\,. \tag{2.10}$$

Hence, using (2.9), (2.10) and the fact that $\mathcal{A}$ is a hermitean operator, we see that

$$\mathcal{A}\, H_n(\alpha x) f(x) = p^n H_n(\alpha x)(f(x))^p\,, \tag{2.11}$$

where

$$\alpha = \sqrt{\frac{1}{2}\frac{p^2-1}{p\ln p}}\,, \tag{2.12}$$

and where $H_n(\xi)$ are the Hermite polynomials with the usual normalization

$$\int_{-\infty}^{+\infty} d\xi \exp(-\xi^2) H_n(\xi) H_m(\xi) = \pi^{1/2} 2^n n! \delta_{nm}\,. \tag{2.13}$$

We now write the $\widetilde{\phi}$ fields in the form

$$\widetilde{\phi}(y,x) = \sum_{n=0}^{\infty} \psi_n(y) 2^{-n/2} H_n(\alpha x) f(x)\,, \tag{2.14}$$

therefore, (2.7) becomes

$$\begin{aligned} S_f = &-\frac{1}{g^2}\frac{p^2}{p-1}\left[\frac{2\pi p^{\frac{2p}{p-1}}\ln p}{p^2-1}\right]^{1/2} \times \\ &\times \int d^{D-1}y \sum_n \frac{n!}{2}\psi_n(y)\left[p^{\left(n-\frac{1}{2}\Box_{\|}\right)} - p\right]\psi_n(y)\,. \end{aligned} \tag{2.15}$$

Hence, the fluctuation modes $\psi_n(x)$ have masses squared given by

$$m^2 = 2(n-1)\,. \tag{2.16}$$

As in the normal bosonic string, the spectrum is discrete with equally spaced levels. The lowest such mode has $n = 0$ and is a tachyon, as was first discussed in [22]. Its mass squared is the same as the effective tachyon mass in (2.1). The next highest mode is at $n = 1$, and is the massless mode discussed in [14].

Let us next consider the interaction terms. To this end we make the substitution

$$\phi(y,x) = f(x) + \sum_n \psi_n(y) 2^{-n/2} H_n(\alpha x) f(x) \tag{2.17}$$

into the interaction term in (2.1). Therefore, the interacting part of the lump action is

$$S_{\mathrm{int}} = \frac{1}{g^2}\frac{p^2}{p-1}\left[\frac{2\pi p^{\frac{2p}{p-1}}\ln p}{p^2-1}\right]^{1/2} \times$$
$$\times \int d^{D-1}y \sum_{\ell=3}^{p+1} \frac{p\,!}{\ell\,!(p+1-\ell)!} \sum_{n_1}\cdots\sum_{n_\ell} A_{n_1 n_2\cdots n_\ell} \prod_{i=1}^{\ell} \psi_{n_i}(y)\,, \quad (2.18)$$

where

$$A_{n_1 n_2\cdots n_\ell} = \left[\frac{2\pi p\ln p}{p^2-1}\right]^{-1/2} \int_{-\infty}^{+\infty} dx \exp\left(-\frac{1}{2}\frac{p^2-1}{p\ln p}x^2\right) \prod_{i=1}^{\ell} 2^{-n_i/2} H_{n_i}(\alpha x)\,. \quad (2.19)$$

To evaluate the expression in (2.19), we note that $H_n(\alpha x)$ can be written in operator form

$$2^{-n/2} H_n(\alpha x) =: (a+a^\dagger)^n :, \quad (2.20)$$

where

$$a = \alpha x + \frac{1}{2\alpha}\frac{\partial}{\partial x}\,. \quad (2.21)$$

Therefore

$$[a, a^\dagger] = 1 \quad (2.22)$$

and the equation in (2.19) becomes[2]

$$A_{n_1 n_2\cdots n_\ell} = \langle 0| \prod_{i}^{\ell} : (a+a^\dagger)^{n_i} : |0\rangle\,, \quad (2.23)$$

where the oscillator vacuum state is normalized to be

$$\langle 0|0\rangle = 1\,. \quad (2.24)$$

Therefore, the coefficient $A_{n_1 n_2\cdots n_\ell}$ essentially counts the number of possible ways to contract the ℓ fields together. To see this more explicitly, let N_{ij} be the number of contractions between $: (a+a^\dagger)^{n_i} :$ and $: (a+a^\dagger)^{n_j} :$ in (2.23). Since the operators are normal ordered $N_{ii} = 0$. The N_{ij} satisfy the ℓ constraint conditions

$$n_i = \sum_{j\neq i} N_{ij} \quad (2.25)$$

and for fixed N_{ij}, the total number of different ways to have this set of contractions is

$$\frac{\prod_i n_i\,!}{\prod_{i<j} N_{ij}\,!}\,. \quad (2.26)$$

[2]The form for $H_n(\alpha x)$ in (2.20) follows from the fact that $: (a+a^\dagger)^n :$ commutes with $x = \frac{1}{2\alpha}(a+a^\dagger)$ and that $: (a+a^\dagger)^n : |0\rangle = (a^\dagger)^n|0\rangle$.

Hence, the coefficients can be expressed as

$$A_{n_1 n_2 \cdots n_\ell} = \sum_{N_{ij}} \frac{\prod_i n_i\,!}{\prod_{i<j} N_{ij}\,!}\,, \tag{2.27}$$

where the sum over all N_{ij} is subject to the constraints in (2.25).

3. p-adic amplitudes for excited modes

In this section we consider the N-point amplitudes for excited p-adic string modes. There are certain points to keep in mind when constructing these amplitudes. First, the number of excited modes that could correspond to fluctuation modes for a tachyon field is much smaller than the number of string modes in ordinary bosonic string theory. This suggests two possibilities. Either an infinite number of fields need to be put in by hand for the full effective field theory, or the number of string modes for p-adic string theory is vastly reduced from ordinary bosonic string theory. The second possibility seems more likely, given the problems previously discussed for gauge fields.

Nevertheless, while massless gauge fields and many other fields might not exist, scattering amplitudes for certain string modes with polarizations transverse to the brane do appear to be consistent. For one thing, these are not gauge fields, so there is no gauge invariance restriction. For a second thing, the existence of tachyon lumps, which follows from the effective action derived from tachyon scattering amplitudes seems to require the existence of such modes.

In [23] it was suggested that for the two derivative truncation of boundary string field theory, the tachyon field condensed to a codimension d brane has fluctuations that correspond to the open string states in (1.2). The vertex operator for such a mode is

$$V = \prod_{i=1}^{d} (\partial_n X^i)^{m_i} e^{ik\cdot X}\,, \tag{3.1}$$

where ∂_n refers to the normal derivative from the boundary of the string worldsheet. It is important to note that k_μ points along the brane coordinates, therefore correlation functions between vertex operators of this sort do not have contributions from the contractions of $\partial_n X^i$ with $e^{ik\cdot X}$ terms.

Let us now specialize to the case where $d = 1$ and let us consider the N-point correlator

$$\left\langle \prod_i^N (\partial X(x_i))^{n_i} e^{ik_i\cdot X(x_i)} \right\rangle . \tag{3.2}$$

Using the fact that

$$\langle X(z_i) X(z_j)\rangle = -\log|z_i - z_j| \tag{3.3}$$

JHEP03(2001)028

we find that the correlator in (3.2) is

$$\sum_{N_{ij}} \prod_i^N n_i! \prod_{i<j} \frac{1}{N_{ij}!} |x_{ij}|^{k_i \cdot k_j - 2N_{ij}} , \tag{3.4}$$

where N_{ij} counts the number of contractions of $\partial X(x_i)$ with $\partial X(x_j)$. As in section 2, the N_{ij} are subject to a constraint

$$n_i = \sum_{j \neq i} N_{ij} . \tag{3.5}$$

Therefore, the N-point amplitude for these excited states is a sum over tachyon like amplitudes, where each term in the sum has the combinatoric prefactor in (2.26) and where $k_i \cdot k_j$ is replaced with $k_i \cdot k_j - 2N_{ij}$. We can then use information about p-adic tachyon amplitudes to find the p-adic excited state amplitudes.

The p-adic amplitudes are made up of a constant term independent of the particles momenta and a sum over terms with poles [20]. The constant term directly corresponds to the N-point interaction term in (2.18). Since, it is independent of $k_i \cdot k_j$, the constant term is unchanged if $k_i \cdot k_j$ is replaced by $k_i \cdot k_j - 2N_{ij}$. Thus each term in the sum in (3.4) has the same constant term. Hence, the general N-point interaction term has an extra factor of

$$\sum_{N_{ij}} \frac{\prod_i^N n_i!}{\prod_{i<j} N_{ij}!} = A_{n_1 n_2 \cdots n_N} , \tag{3.6}$$

precisely matching the result of the field theory analysis.

To complete the proof, we need to show that the poles in the amplitude derived from the correlator in (3.2) factorize consistently with the action in (2.15) and (2.18). We proceed using an induction argument. If we isolate the amplitude to a particular pole term, then the propagator divides the Feynman diagram into two parts (see figure 1).

Let us assume that there are M vertices on one side of the propagator and $L = N - M$ vertices on the other side. By assumption, the part of the amplitude containing this pole factorizes to the form

$$\begin{aligned} & A\Big(n_1, k_1; n_2, k_2; \ldots n_M, k_M; n_I, k_I\Big) \frac{1}{n_I!} \frac{1}{p^{n_I + k_I^2/2} - p} \times \\ & \times A\Big(n_I, k_I; n_{M+1}, k_{M+1} \ldots n_N, k_N\Big) , \end{aligned} \tag{3.7}$$

where $A(n_1, k_1; n_2, k_2; \ldots n_M, k_M; n_I, k_I)$ and $A(n_I, k_I; n_{M+1}, k_{M+1} \ldots n_N, k_N)$ refer to the lower point amplitudes derived from (2.15) and (2.18) and n_I and k_I refer to the mode number and momentum of the internal state. Hence the momentum going through the propagator is

$$k_I = \sum_i^M k_i = - \sum_{i=M+1}^N k_i , \tag{3.8}$$

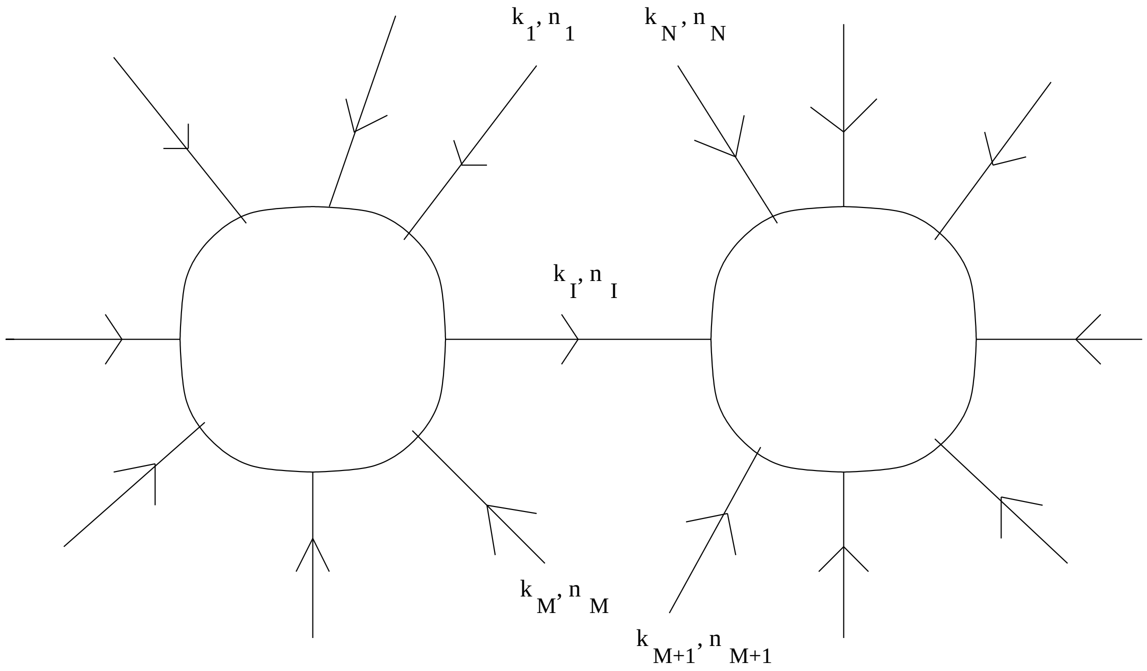

Figure 1: Factorization of the amplitude with incoming momenta k_i and mode number n_i about a pole with momentum k_I and mode number n_I.

and so

$$\begin{aligned} k_I^2 &= \sum_i^M k_i^2 + 2\sum_{i<j}^M k_i \cdot k_j = 2\sum_i^M (1-n_i) + 2\sum_{i<j}^M k_i \cdot k_j \\ &= 2M + 2\sum_{i<j}^M (k_i \cdot k_j - 2M_{ij}) - 2\sum_i^M \sum_j^L \widetilde{N}_{ij}\,, \end{aligned} \tag{3.9}$$

where

$$\begin{aligned} M_{ij} &\equiv N_{ij}\,, \qquad i,j \le M\,, \\ \widetilde{N}_{ij} &\equiv N_{i,j+M}\,, \qquad i \le M, \qquad j \le L\,. \end{aligned} \tag{3.10}$$

Comparing to a pure tachyon amplitude, which only has tachyon poles, it is clear that the mass squared of the pole is given by

$$m_I^2 = -2 + 2\sum_i^M \sum_j^L \widetilde{N}_{ij}\,, \tag{3.11}$$

and so the mode number for the pole is

$$n_I = \sum_i^M \sum_j^L \widetilde{N}_{ij}\,. \tag{3.12}$$

By momentum conservation, we also have that

$$k_I^2 = 2L + 2\sum_{i<j}^{L}(k_i \cdot k_j - 2L_{ij}) - 2\sum_{i}^{M}\sum_{j}^{L}\widetilde{N}_{ij}\,, \tag{3.13}$$

where $L_{ij} \equiv N_{i+M,j+M}$, hence we have the relation that

$$M + \sum_{i<j}^{M}(k_i \cdot k_j - 2M_{ij}) = L + \sum_{i<j}^{L}(k_i \cdot k_j - 2L_{ij})\,. \tag{3.14}$$

There are $N(N-1)/2$ separate N_{ij} terms. However, the constraint in (3.5) reduces the number of independent terms to $N(N-3)/2$. Since we are isolating the amplitude to a particular pole, n_I is fixed, reducing the number of independent terms by 1. Finally, fixing all M_{ij} and L_{ij} reduces the number of independent N_{ij} terms by $(M+1)M/2-(M+1)$ and $(L+1)L/2-(L+1)$, leaving $(M-1)(L-1)$ independent terms. These independent terms are the $\widetilde{N}_{ij}$, subject to the constraints

$$M_{iI} \equiv n_i - \sum_{k\neq i} M_{ik} = \sum_k \widetilde{N}_{ik}\,, \qquad L_{jI} \equiv \widehat{n}_j - \sum_{k\neq j} L_{jk} = \sum_k \widetilde{N}_{kj}\,, \tag{3.15}$$

where $\widehat{n}_i = n_{i+M}$. The terms M_{iI} and L_{iI} count the contractions of the external states with the internal state, and impose $M+L-1$ constraints.

We now write the combinatoric factor in (2.26) as

$$\frac{\prod_i n_i!}{\prod_{i<j} N_{ij}!} = \frac{\prod_i^M n_i! \prod_i^L \widehat{n}_i!}{\prod_{i<j}^M M_{ij}! \prod_{i<j}^L L_{ij}! \prod_{i,j} \widetilde{N}_{ij}!}\,. \tag{3.16}$$

Since $\widetilde{N}_{ij}$ enters into the factorized amplitudes only through the M_{iI} and L_{jI} terms, the complete factorized amplitude should consist of a sum over all $(M-1)(L-1)$ independent $\widetilde{N}_{ij}$. It is convenient to use as the independent terms $\widetilde{N}_{ij}$ with $i<M$ and $j<L$. Then the dependent terms can be written as

$$\begin{aligned}
\widetilde{N}_{iL} &= n_i - \sum_k^{L-1}\widetilde{N}_{ik} - \sum_k M_{ik}\,,\\
\widetilde{N}_{Mj} &= \widehat{n}_j - \sum_k^{M-1}\widetilde{N}_{kj} - \sum_k L_{kj}\,,\\
\widetilde{N}_{ML} &= n_M - \sum_k^{L-1}\widetilde{N}_{Mk} - \sum_k M_{Mk}\\
&= n_M - \sum_k^{L-1}\widehat{n}_k + \sum_m^{M-1}\sum_k^{L-1}\widetilde{N}_{mk} - \sum_k M_{Mk} + \sum_{jk} L_{jk}\,.
\end{aligned} \tag{3.17}$$

Thus, using this way to choose the dependent terms, we see that the factor in (3.16) dependent on $\widetilde{N}_{ij}$ is

$$\frac{1}{\widetilde{N}_{ij}!\widetilde{N}_{Mj}!\widetilde{N}_{iL}!\widetilde{N}_{ML}!}\,. \tag{3.18}$$

Starting with the sum over $\widetilde{N}_{11}$, we have

$$\sum_{\widetilde{N}_{11}} \frac{1}{\widetilde{N}_{11}!\widetilde{N}_{M1}!\widetilde{N}_{1L}!\widetilde{N}_{ML}!} = \sum_{\widetilde{N}_{11}} \binom{\widetilde{N}_{11}+\widetilde{N}_{1L}}{\widetilde{N}_{11}} \binom{\widetilde{N}_{M1}+\widetilde{N}_{ML}}{\widetilde{N}_{1L}} \times$$
$$\times \frac{1}{(\widetilde{N}_{11}+\widetilde{N}_{1L})!(\widetilde{N}_{M1}+\widetilde{N}_{ML})!}\,. \tag{3.19}$$

Note that $(\widetilde{N}_{11}+\widetilde{N}_{1L})$ and $(\widetilde{N}_{M1}+\widetilde{N}_{ML})$ on the r.h.s. of (3.19) are independent of $\widetilde{N}_{11}$. Using the identity

$$\sum_m \binom{N_1}{m}\binom{N_2}{N_3-m} = \binom{N_1+N_2}{N_3}, \tag{3.20}$$

we can write (3.19) as

$$\frac{(\widetilde{N}_{11}+\widetilde{N}_{1L}+\widetilde{N}_{M1}+\widetilde{N}_{ML})!}{(\widetilde{N}_{11}+\widetilde{N}_{M1})!(\widetilde{N}_{1L}+\widetilde{N}_{ML})!(\widetilde{N}_{11}+\widetilde{N}_{1L})!(\widetilde{N}_{M1}+\widetilde{N}_{ML})!}\,. \tag{3.21}$$

The last two terms in the denominator of this expression have $\widetilde{N}_{12}$ dependence. Hence, after dividing (3.21) by $\widetilde{N}_{12}!\widetilde{N}_{M2}!$ and using (3.20), the sum over $\widetilde{N}_{12}$ gives

$$\frac{(\widetilde{N}_{11}+\widetilde{N}_{12}+\widetilde{N}_{1L}+\widetilde{N}_{M1}+\widetilde{N}_{M2}+\widetilde{N}_{ML})!}{(\widetilde{N}_{11}+\widetilde{N}_{M1})!(\widetilde{N}_{12}+\widetilde{N}_{M2})!(\widetilde{N}_{1L}+\widetilde{N}_{ML})!(\widetilde{N}_{11}+\widetilde{N}_{12}+\widetilde{N}_{1L})!(\widetilde{N}_{M1}+\widetilde{N}_{M2}+\widetilde{N}_{ML})!}\,. \tag{3.22}$$

Following this same procedure for the sums of all $\widetilde{N}_{1j}$, $j<M$, results in the factor

$$\frac{\left(\sum_j^L(\widetilde{N}_{1j}+\widetilde{N}_{Mj})\right)!}{\left(\sum_j^L \widetilde{N}_{1j}\right)!\left(\sum_j^L \widetilde{N}_{Mj}\right)!\prod_j^L(\widetilde{N}_{1j}+\widetilde{N}_{Mj})!}\,. \tag{3.23}$$

One can now continue in a similar fashion for all sums, where one uses the identity (3.20) for each sum. At the end, one finds that

$$\sum_{N_{ij}}\prod_i^M\prod_j^L \frac{1}{\widetilde{N}_{ij}!} = \frac{\left(\sum_{ij}\widetilde{N}_{ij}\right)!}{\prod_i\left(\sum_j \widetilde{N}_{ij}\right)!\prod_j\left(\sum_i \widetilde{N}_{ij}\right)!} = \frac{n_I!}{\prod_i M_{iI}!\prod_j L_{jI}!}\,, \tag{3.24}$$

where we used eqs. (3.12) and (3.15) for the last equality. Thus, the amplitudes

JHEP03(2001)028

$A(n_1,k_1;n_2,k_2;\dots n_M,k_M;n_I,k_I)$ and $A(n_I,k_I;n_{M+1},k_{M+1}\dots n_N,k_N)$ have combinatoric factors

$$\frac{\prod_i^M n_i!}{\prod_{i<j}^M M_{ij}!}\frac{n_I!}{\prod_i^M M_{iI}!} \tag{3.25}$$

and

$$\frac{\prod_i^L \widehat{n}_i!}{\prod_{i<j}^L L_{ij}!}\frac{n_I!}{\prod_i^L L_{iI}!}\,, \tag{3.26}$$

respectively. Therefore, each part of the factorized amplitude has the same form for its combinatoric factors as the original amplitude. We can then continue factorizing poles until we are left with pole free amplitudes. These amplitudes then have the combinatoric factors in (3.6). This completes the proof.

4. Discussion

We have shown that the effective field theory for the tachyon fluctuations about a p-adic lump is consistent with tree-level p-adic string amplitudes. This analysis is further evidence for the identification of tachyon fluctuations with specific open string states. The analysis in [23] went further, conjecturing the identification of gauge fluctuations with other open string states. The apparent absence of gauge fields prevents us from testing this in the p-adic case.

There also have been attempts to generalize the closed string by replacing integrals over the complex plane with integrals over p-adics, or more precisely, with integrals over extensions of p-adic numbers [15]–[18], [25]. The resulting amplitudes are almost identical to the open string amplitudes, and in fact an effective field theory was found for the tachyon, which is identical to the action in (2.1), except that the d'Alembertian has a factor of 1/4 in front of it instead of 1/2 [25]. But this means that this action has lump solutions with constant descent relations. If this is really a closed string theory, then it seems difficult to interpret the lumps as analogs of D-branes. A possible explanation is that this construction is the analog of the open string for *extended* p-adic fields.

If this is the case, then it begs the question of whether there is a p-adic closed string. The apparent absence of gauge fields on the p-adic worldsheet might provide a clue. It has been argued that in ordinary string theory, the closed strings are flux tubes coming from these gauge fields [24], [26]–[31]. But if the gauge fields are absent, then there might not be any closed strings to be concerned about.

Acknowledgments

I thank Barton Zwiebach for many helpful discussions and for critically reading the manuscript. I also thank the CTP at MIT for hospitality during the course of this work. This work was supported in part by the NFR.

References

[1] A. Sen, *Descent relations among bosonic D-branes*, *Int. J. Mod. Phys.* **A 14** (1999) 4061 [hep-th/9902105]; *Stable non-BPS bound states of BPS D-branes*, *J. High Energy Phys.* **08** (1998) 010 [hep-th/9805019]; *Tachyon condensation on the brane antibrane system*, *J. High Energy Phys.* **08** (1998) 012 [hep-th/9805170]; SO(32) *spinors of type-I and other solitons on brane-antibrane pair*, *J. High Energy Phys.* **09** (1998) 023 [hep-th/9808141].

[2] V.A. Kostelecky and S. Samuel, *On a nonperturbative vacuum for the open bosonic string*, *Nucl. Phys.* **B 336** (1990) 263;
A. Sen and B. Zwiebach, *Tachyon condensation in string field theory*, *J. High Energy Phys.* **03** (2000) 002 [hep-th/9912249];
W. Taylor, *D-brane effective field theory from string field theory*, *Nucl. Phys.* **B 585** (2000) 171 [hep-th/0001201];
N. Moeller and W. Taylor, *Level truncation and the tachyon in open bosonic string field theory*, *Nucl. Phys.* **B 583** (2000) 105 [hep-th/0002237];
J.A. Harvey and P. Kraus, *D-branes as unstable lumps in bosonic open string field theory*, *J. High Energy Phys.* **04** (2000) 012 [hep-th/0002117];
R. de Mello Koch, A. Jevicki, M. Mihailescu and R. Tatar, *Lumps and p-branes in open string field theory*, *Phys. Lett.* **B 482** (2000) 249 [hep-th/0003031];
N. Moeller, A. Sen and B. Zwiebach, *D-branes as tachyon lumps in string field theory*, *J. High Energy Phys.* **08** (2000) 039 [hep-th/0005036];
R. de Mello Koch and J.P. Rodrigues, *Lumps in level truncated open string field theory*, *Phys. Lett.* **B 495** (2000) 237 [hep-th/0008053];
N. Moeller, *Codimension two lump solutions in string field theory and tachyonic theories*, hep-th/0008101;
A. Sen and B. Zwiebach, *Large marginal deformations in string field theory*, *J. High Energy Phys.* **10** (2000) 009 [hep-th/0007153];
W. Taylor, *Mass generation from tachyon condensation for vector fields on D-branes*, *J. High Energy Phys.* **08** (2000) 038 [hep-th/0008033].

[3] N. Berkovits, *The tachyon potential in open Neveu-Schwarz string field theory*, *J. High Energy Phys.* **04** (2000) 022 [hep-th/0001084];
N. Berkovits, A. Sen and B. Zwiebach, *Tachyon condensation in superstring field theory*, *Nucl. Phys.* **B 587** (2000) 147 [hep-th/0002211];
P.-J.D. Smet and J. Raeymaekers, *Level four approximation to the tachyon potential in superstring field theory*, *J. High Energy Phys.* **05** (2000) 051 [hep-th/0003220];
A. Iqbal and A. Naqvi, *Tachyon condensation on a non-BPS D-brane*, hep-th/0004015; *On marginal deformations in superstring field theory*, *J. High Energy Phys.* **01** (2001) 040 [hep-th/0008127].

[4] A. Sen, *Universality of the tachyon potential*, *J. High Energy Phys.* **12** (1999) 027 [hep-th/9911116];
L. Rastelli and B. Zwiebach, *Tachyon potentials, star products and universality*,

hep-th/0006240;
V.A. Kostelecky and R. Potting, *Analytical construction of a nonperturbative vacuum for the open bosonic string*, *Phys. Rev.* **D 63** (2001) 046007 [hep-th/0008252];
H. Hata and S. Shinohara, *Brst invariance of the non-perturbative vacuum in bosonic open string field theory*, *J. High Energy Phys.* **09** (2000) 035 [hep-th/0009105];
B. Zwiebach, *Trimming the tachyon string field with* SU(1, 1), hep-th/0010190.

[5] L. Rastelli, A. Sen and B. Zwiebach, *String field theory around the tachyon vacuum*, hep-th/0012251.

[6] J.A. Minahan and B. Zwiebach, *Field theory models for tachyon and gauge field string dynamics*, *J. High Energy Phys.* **09** (2000) 029 [hep-th/0008231].

[7] B. Zwiebach, *A solvable toy model for tachyon condensation in string field theory*, *J. High Energy Phys.* **09** (2000) 028 [hep-th/0008227].

[8] E. Witten, *Some computations in background independent off-shell string theory*, *Phys. Rev.* **D 47** (1993) 3405 [hep-th/9210065].

[9] A.A. Gerasimov and S.L. Shatashvili, *On exact tachyon potential in open string field theory*, *J. High Energy Phys.* **10** (2000) 034 [hep-th/0009103].

[10] D. Kutasov, M. Mariño and G. Moore, *Some exact results on tachyon condensation in string field theory*, *J. High Energy Phys.* **10** (2000) 045 [hep-th/0009148].

[11] D. Ghoshal and A. Sen, *Normalisation of the background independent open string field theory action*, *J. High Energy Phys.* **11** (2000) 021 [hep-th/0009191].

[12] J.A. Minahan and B. Zwiebach, *Effective tachyon dynamics in superstring theory*, hep-th/0009246.

[13] D. Kutasov, M. Marino and G. Moore, *Remarks on tachyon condensation in superstring field theory*, hep-th/0010108.

[14] D. Ghoshal and A. Sen, *Tachyon condensation and brane descent relations in p-adic string theory*, *Nucl. Phys.* **B 584** (2000) 300 [hep-th/0003278].

[15] P.G. O. Freund and M. Olson, *Nonarchimedean strings*, *Phys. Lett.* **B 199** (1987) 186.

[16] P.G.O. Freund and E. Witten, *Adelic string amplitudes*, *Phys. Lett.* **B 199** (1987) 191.

[17] I.V. Volovich, *P-adic string*, *Class. and Quant. Grav.* **4** (1987) L83.

[18] B. Grossman, *P-adic strings, the Weyl conjectures and anomalies*, *Phys. Lett.* **B 197** (1987) 101.

[19] P.H. Frampton and Y. Okada, *The p-adic string N point function*, *Phys. Rev. Lett.* **60** (1988) 484.

[20] L. Brekke, P.G. O. Freund, M. Olson and E. Witten, *Nonarchimedean string dynamics*, *Nucl. Phys.* **B 302** (1988) 365.

[21] P.H. Frampton and Y. Okada, *Effective scalar field theory of p-adic string*, *Phys. Rev.* **D 37** (1988) 3077.

[22] P.H. Frampton and H. Nishino, *Stability analysis of p-adic string solitons*, *Phys. Lett.* **B 242** (1990) 354.

[23] J.A. Minahan and B. Zwiebach, *Gauge fields and fermions in tachyon effective field theories*, *J. High Energy Phys.* **02** (2001) 034 [hep-th/0011226].

[24] M. Kleban, A.E. Lawrence and S. Shenker, *Closed strings from nothing*, hep-th/0012081.

[25] P.H. Frampton and H. Nishino, *Theory of p-adic closed strings*, *Phys. Rev. Lett.* **62** (1989) 1960.

[26] P. Yi, *Membranes from five-branes and fundamental strings from Dp-branes*, *Nucl. Phys.* **B 550** (1999) 214 [hep-th/9901159].

[27] O. Bergman, K. Hori and P. Yi, *Confinement on the brane*, *Nucl. Phys.* **B 580** (2000) 289 [hep-th/0002223].

[28] J.A. Harvey, P. Kraus, F. Larsen and E.J. Martinec, *D-branes and strings as non-commutative solitons*, *J. High Energy Phys.* **07** (2000) 042 [hep-th/0005031].

[29] G. Gibbons, K. Hori and P. Yi, *String fluid from unstable D-branes*, *Nucl. Phys.* **B 596** (2001) 136 [hep-th/0009061].

[30] A. Sen, *Fundamental strings in open string theory at the tachyonic vacuum*, hep-th/0010240.

[31] A.A. Gerasimov and S.L. Shatashvili, *Stringy Higgs mechanism and the fate of open strings*, *J. High Energy Phys.* **01** (2001) 019 [hep-th/0011009].